Fundamental Mathematics for Robotics
Homework Set #09, Dr.T

[1] Suppose that an angular speed of a joint is given by the following equation:

$$\omega(t) = 3 - 2\text{t}, \quad 0 \le t$$

Also suppose that the joint is at the origin, i.e., $\theta(0) = 0$ until time $t = 0$.

(a) Write $\theta(2)$ in the form of the Riemann sum with $the\ number\ of\ intervals = N$.
(b) Compute $\theta(2)$ using the Riemann sum at $T = 0.2$ ($or\ N = 10$).
(c) Repeat (b) with $T = 1, 0.1, \text{and } 0.01$.
(d) Compute $\theta(2)$ by taking the limit of $T \to 0$ (or $N \to \infty$) of the Riemann sum.

[2] Repeat Problem [1] with $\omega(t) = t^2 - 2\text{t}, \quad 0 \le t$.

[3] (Extra) How should we modify the solution of Problem [1] if the joint is initially at $\theta(0) = 10$?

[4] Discuss the following issues. The summation in the Riemann sum is up to $N - 1$. Why this is not N?

[5] Prove the following: $\text{S} = \sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

[6] (Extra) Prove the following: $\text{S} = \sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

[7] (Extra) Suppose that the angle $\theta(t)$ and the angular speed $\omega(t)$ were 0 at time $t = 0$ and that the angular acceleration $\dot{\omega}(t)$ is given by the following:

$$\dot{\omega}(t) = 3, \quad 0 \le t \le 2$$

(d) Compute the angular speed at time 2, i.e., $\omega(2)$ using the time interval $T = 0.5$.
(e) Compute the angle at time 2, i.e., $\theta(2)$ using the time interval $T = 0.5$.
(f) Repeat parts (a) and (b) using the time interval T=0.05.