Fundamental Mathematics for Robotics
Homework Set #07-2, Dr.T

Note: Turn in your solutions by the start of the Recitation class on June 9, 2015.

[1] Evaluate the derivative of the following functions using the shortcut Laws. Remember that you can use up to two Laws in a single step and you must state which Laws are used.

(a) $f(x) = (x-2)^2$
(b) $x(t) = \cos(3t-4)$
(c) $y(t) = 3\cos(4t+3) + 2\ \sin(-2t+1)$
(d) $z(t) = 3\ \sin(-t^2+3t-2)$
(e) $g(y) = e^{-y+3} + y^2$
(f) $h(z) = ze^{2z+3}$
(g) (Extra) $g(t) = 2\cos\{\theta(t)\}$
(h) (Extra) $h(x) = e^{-e^{2x+3}}$

[2] Find the derivative of the following function using the definition of the differentiation. Hint: You can use the following equalities:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1, \qquad \lim_{x\to 0}\frac{1-e^{-x}}{x} = 1$$

(a) $f(\theta) = \sin 2\theta$
(b) $f(t) = e^{-t}$
(c) (Extra) $f(\theta) = \cos\theta$

[3] The two definitions of average speed at time t over the interval Δt shown below are, in fact, equivalent when computing the instantaneous speed. Confirm this by computing the instantaneous angular speed $d\theta/dt$ for $\theta(t) = (t+1)^3$ using both definitions.

$$\frac{\theta(t+\Delta t)-\theta(t)}{\Delta t} \text{ and } \frac{\theta(t)-\theta(t-\Delta t)}{\Delta t}$$

[4] (Extra) (This problem is designed to relax your brain. Enjoy!)

(a) Show that if $r = 1/(r-1)$, then $q = 1/(q+1)$ for $q = 1/r$. Also, find r and q.
(b) Find the number x: $x = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{\cdots}}}}$ (Continued fraction)

Fundamental Mathematics for Robotics
Extra Homework Set #07-Extra1, Dr.T

[1] Compute the derivative of the following function using the definition of the differentiation.

(a) $f(t) = -2t^2 + 3t - 4$
(b) $g(t) = \frac{3}{t+2}$
(c) $v(t) = \sqrt{3-t}$
(d) $w(t) = (t-2)^3$
(e) $y(t) = 3t\sqrt{t}$
(f) (Extra) $x(t) = \frac{2}{\sqrt{t}+3}$

[2] Evaluate the derivative of the following function using the definition of the differentiation.

(a) $f(t) = -2(2t+3)^2 + 3(2t+3) - 4$
(b) $x(t) = \frac{t}{t+3}$
(c) $v(t) = \sqrt{t^2+3}$
(d) $\psi(t) = \frac{4}{5+t^2}$
(e) $\omega(t) = \frac{2}{\sqrt{t+3}}$
(f) (Extra) $\phi(t) = \frac{t}{\sqrt{1-4t^2}}$

[3] Evaluate the derivative of the following function using the shortcut Laws. Remember that you can use up to two Laws in one step.

(a) $\theta(t) = \sin 3t \cdot \cos 4t$
(b) $x(t) = \mathrm{e}^{-4t}\cos(3t)$
(c) $\theta(t) = 3t\sin(-2t+3)$
(d) $y(t) = t^2\cos(-2t^2+3t)$
(e) $\theta(t) = t^2\mathrm{e}^{4t}$
(f) $\varphi(t) = t^2\mathrm{e}^{4t^2}$
(g) $v(t) = 2t^2\sqrt{2-t}$
(h) $\phi(t) = \frac{2t+6}{(t-2)^2}$
(i) $w(t) = \frac{t}{\sqrt{1-4t^2}}$
(j) $\psi(t) = \frac{4}{3+\cos(3t)}$
(k) (Extra) $z(t) = 5t\mathrm{e}^{4t}\cos(3t-2)$

Fundamental Mathematics for Robotics
Extra Homework Set #07-Extra2, Dr.T

[1] Find the indicated derivative of the given function.

(a) $f(x) = 3x^4 - 2x^3 + x^2 - 2x + 4, \quad f^{(3)}(x)$

(b) $y(x) = \frac{1}{\sqrt{x}}, \quad y^{(4)}(x)$

(c) $g(t) = \sin 3t + \cos 2t, \quad g^{(2)}(t)$

(d) $\theta(t) = \cos 2t \sin 3t, \quad \ddot{\theta}(t)$

(e) (Extra) $h(t) = e^{-t}\sin 2t, \quad \ddot{h}(t)$

[2] Let us consider differentiating a quotient $f(x)/g(x)$.

(a) Differentiate $h(x) = 1/g(x)$ using the chain rule.

(b) Differentiate $p(x) = f(x)h(x)$ using Law (d).

(c) Obtain Law (h) for Differentiation of Quotient using the results in part (a) and (b).

[3] Let us consider exponential functions with a general base α.

(a) Show that $\alpha^x = e^{\{ln(\alpha)\}x}$. Hint: $y = e^x \Leftrightarrow x = ln\, y$.

(b) Use (a) to compute the derivative of $f(x) = \alpha^x$.

(c) Plot $f(x) = \alpha^x$ for various values of α, α = 1, 1.5, 2, 2.5, e, 3, 5, 10 over the interval [-2, 2]. Hint: Use [1, 10] for the range of y-axis.

(d) Plot $f(x)$ and its derivative pairwise for the α values given in Part (c) over the interval [-2, 2]. Discuss how the relationship between the function and the derivative changes.

[4] (Extra) (This problem is designed to relax your brain. Enjoy!)

(a) Can you compute the number x given by $x = \sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}$, an infinitely repeated power? (Hint1: Look at the pattern! Hint2: What should be the answer to the question starting with 'Can you ...?')