Fundamental Mathematics for Robotics
Homework Set #05-2, Dr.T

[1] Let us consider numerical examples of a two-link manipulator given in the lecture. Let us use the following values: $L_1 = 50(cm), L_2 = 30(cm)$. Compute the end-effector (EF) position (x_2, y_2) for the following joint angles.

(a) $\theta_1 = 30(deg), \theta_2 = 60(deg)$
(b) $\theta_1 = 60(deg), \theta_2 = 30(deg)$
(c) $\theta_1 = -30(deg), \theta_2 = 60(deg)$
(d) $\theta_1 = 30(deg), \theta_2 = -60(deg)$
(e) $\theta_1 = -13.57(deg), \theta_2 = 60(deg)$
(f) What is the relationship between (d) and (e)?

[2] Let us fill in derivations that were skipped in the lecture. From the 2-Link equation:

$x_2 = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$ (1)

$y_2 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$ (2)

(a) Find $A_1, A_2, B_1, and\ B_2$ in $(1) \rightarrow x_2 = A_1 \sin\theta_1 + A_2 \cos\theta_1$ (3)
$(2) \rightarrow y_2 = B_1 \sin\theta_1 + B_2 \cos\theta_1$ (4)

(b) Derive $C_1 \sin\theta_1 + C_2 \cos\theta_1 = 0$ by eliminating l.h.s. from (3) and (4).

(c) Show that $\theta_1 = \tan^{-1}\frac{-C_2}{C_1}$.

[3] Let us do the Inverse-Kinematics computation for the 2-Link manipulator in Problem [1]. Can you compute θ_1, and θ_2 from the following EF positions (x_2, y_2).

(a) $x_2 = 73.3, y_2 = 25$
(b) $x_2 = 58.3, y_2 = 50.98$
(c) $x_2 = 10.0, y_2 = 69.28$
(d) $x_2 = 65.36, y_2 = 35.36$
(e) $x_2 = 60.0, y_2 = 60.0$

[4] Solve the following questions on trigonometric functions using inverse trigonometric functions. Use the region $-\pi < \theta \leq \pi$. You must specify the quadrant in which your answer is located.

(a) Solve for the angle θ: $5\cos\theta + 3 = 0$
(b) Solve for the angle θ: $5\sin\theta + 4 = 0$
(c) Use ATAN2(x,y) to solve for the angle θ: $4\cos\theta + 3\sin\theta = 0$ (Hint: Which quadrant?)
(d) Prove: $\cos^2\theta = 1/(1 + \tan^2\theta)$
(e) (Extra) Use the result of (d) for (a) to use ATAN2 to solve for the angle θ.