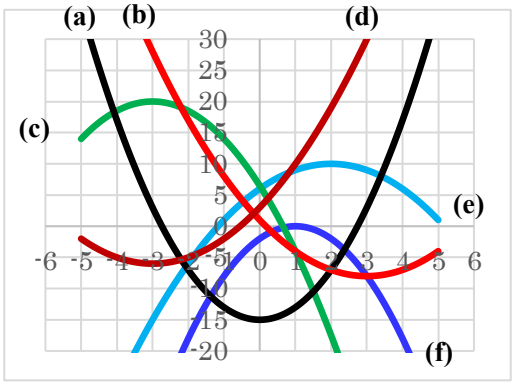
Fundamental Mathematics for Robotics
Homework Set #05-1, Dr.T

[1] The given figure shows graphs of $f(x) = Ax^2 + Bx + C$. Find $A, B,$ and C of each graph.



[2] Plot or sketch the functions $\sin\theta$ and $\cos\theta$ for $-2\pi \le \theta \le 2\pi$ on a single graph.

[3] Plot or sketch the following functions:
(a) $\cos 2\theta$ for $-\pi \le \theta \le \pi$
(b) $-\cos 2\theta$ for $-\pi \le \theta \le \pi$
(c) $1 - \cos 2\theta$ for $-\pi \le \theta \le \pi$
(d) $\frac{1}{2}(1 - \cos 2\theta)$ for $-\pi \le \theta \le \pi$
(e) Discuss what the resulting graph is.

[4] Plot or sketch the following functions:
(a) $f(x) = \frac{x^2+3}{x^2+1}$ on $[-10, 10]$
(b) $g(x) = \begin{cases} -x^2 + 3, & x < 1 \\ x^2 - 4x + 5, & 1 < x \end{cases}$ on $[-1, 3]$

[5] Plot or sketch the exponential function e^{at+b} for $a = -1, 0, 1$ *and* $b = -1, 0, 1$ over the interval [–2, 2] on a single graph. (How many curves do you need to plot or sketch?)

[6] Find two functions $f(x)$ that satisfy all of the following conditions. One of them should contain exponential functions only (you can use all four arithmetic operations with constants).
i. $f(x)$ is continuous.
ii. $f(x)$ goes to 0 as x goes to ∞.
iii. $f(x)$ goes to 1 as x goes to $-\infty$.
iv. $f(x)$ is monotone decreasing.

[7] Use the Euler's formula to prove the double angle formula given by

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

[8] (Extra) Use the Euler's formula to prove the following formula:

$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$