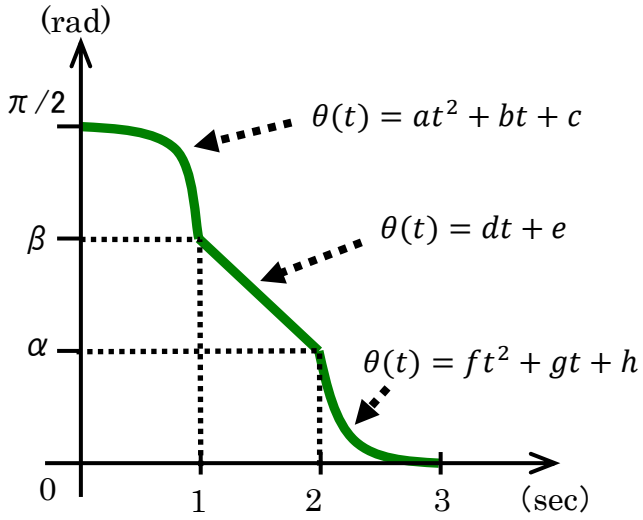
Fundamental Mathematics for Robotics
Homework Set #04, Dr.T

[1] Let us repeat the Robo-Kick problem with different set-up. Suppose that the given figure is a graph of the angle $\theta(t)$ of the knee joint, where **a**, **b**, **c**, **d**, **e**, **f**, **g**, and **h** are parameters and α and β are adjustable angles.



(a) The graph of $\theta(t)$ shows both the initial and final conditions, $\theta(0) = \pi/2$ (rad) and $\theta(3) = 0$ (rad). Use them to find two equations.

(b) The joint angle must be continuous at time $t = 1$ and $t = 2$ (sec). Use $t-$ and $t+$ notations to express those constraints and write two equations.

(c) The graph also shows that the values of the joint angle $\theta(t)$ is β (rad) at $t = 1$ (sec) and α (rad) at $t = 2$ (sec). Use this information to write two equations.

(d) The joint was not moving before time $t = 0$ (sec) and this implies that the angular speed at $t = 0$ (sec) is zero. Use this constraint to write an equation.

(e) It seems that the slope of the graph at $t = 3$ (sec) is zero, namely, the knee joint stops. Use this constraint to write an equation.

(f) Find **a**, **b**, **c**, **d**, **e**, **f**, **g** and **h** from the above conditions using adjustable angles α and β.

(g) Choose several (at least three) pairs of α and β and plot (using a PC) or sketch (handwriting) the angle $\theta(t)$ for those values of α and β where

$$0 < \alpha < \beta < \pi/2$$

(h) (Extra) Suppose we want to make the trajectory 'smooth,' meaning that the speed is also continuous, i.e., the derivative $d\theta(t)/dt$ is continuous all the time. Write two equations of continuity conditions for the derivative at times $t = 1$ and 2 (sec). Find the values of α and β that realizes the continuity.

(i) (Extra) what should be the units of parameters **a** through **h**? Hint: The unit of t is (sec) and that of $\theta(t)$ is (rad).

[2] Let's design original trajectories for the knee joint of a robot. We keep the total time interval of 3 (sec), the initial angle of $\pi/2$ (rad), the final angle of 0 (rad), and the initial speed of 0 (rad/sec). You can change

(1) the number of subintervals,
(2) the lengths of subintervals, and
(3) the types of functions used in subintervals.

Make sure you plot or sketch your trajectories. Hint: In Problem [1], we used 3 subintervals, equal subinterval length of 1 (sec), and polynomials, respectively.