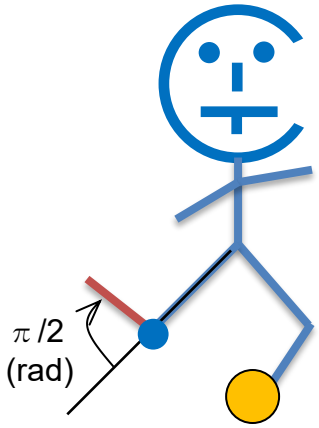
Fundamental Mathematics for Robotics
Homework Set #03, Dr.T

[1] Suppose that the motion of the knee joint for Robo-Kick 1 is needed. Here the initial and the final conditions are $\theta(0) = \pi/2$ (rad), $\dot{\theta}(0) = 0, \theta(3) = 0$(rad), $\dot{\theta}(3) = 0$. Design a trajectory following what we learned in the lecture. (Several notes are due here. The direction of the knee joint is taken clockwise because the range of motion of the knee joint is lopsided and it make sense to make the direction of motion to be positive. It is also noted that rad (radian) is used more often in robotics.)



[2] Let us try a different set of assumptions for Robo-Kick 2.
1. The direction of measuring the hip joint is reversed.
2. Let the initial condition be $\theta(0) = \pi/3$ (rad), $\dot{\theta}(0) = 0$.
3. Let the final condition be $\theta(3) = -\pi/6$ (rad), $\dot{\theta}(3) = 0$.

Find a trajectory in the form of a 3rd order polynomial and draw the trajectory and the speed.

[3] Let us try a different function for Robo-Kick 2. Since the graph of the resulting 3rd order polynomial looks like a half of one period of cosine function, how about trying $\theta(t) = A + B\cos Ct$, where $A, B,$ and C are parameters.

(a) Find the parameter values using four constraints given in the class, namely $\theta(0) = -60$(deg), $\dot{\theta}(0) = 0, \theta(3) = 30$(deg), $\dot{\theta}(3) = 0$.
(b) Plot or sketch the trajectory you found in (a).
(c) Compute and plot (or sketch) the speed of the trajectory.
(d) (Extra) Compute and plot (or sketch) the acceleration of the trajectory.
(e) Does the number of parameters match the number of constraints? Obviously not. Explain why we can find an answer? (A hint is in the problem statement.)

[4] Let us find the units of the quantities appearing the following equation, where $f(t)$ is in kg·m/sec^2 and $\theta(t)$ is in rad:

$$f(t) = M\ddot{\theta}(t) + D\dot{\theta}(t) + K\theta(t)$$