Fundamental Mathematics for Robotics
Homework Set #13, Dr.T

[1] Show the following Maclaurin expansion.

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \quad \text{for} \quad |x| < 1$$

Here, ignore the region of convergence $|x| < 1$.

[2] Let us show the Maclaurin series expansion of the function $f(x) = 1/(1-x)$ following (a) through (c).

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \quad \text{for} \quad |x| < 1$$

(a) Show $f_n(x) = 1 + x + x^2 + x^3 + \cdots + x^n = \frac{1-x^{n+1}}{1-x}$.

(b) Show the given expansion using $\lim_{n\to\infty} f_n(x) = f(x)$.

(c) Find the condition for the convergence of the series by observing the condition for the convergence of the limit in (b).

[3] (Extra) The convergence of a power series can be checked by the d'Alembert's test for convergence given by the following: A series $\sum_{n=0}^{\infty} a_n$ converges if

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$$

Use this test to show the range of convergence $|x| < 1$ for x in Problem [1].

[4] Use the d'Alembert's test for convergence to show that the range of convergence for the power series expansion of the exponential function

$$e^{ax} = \sum_{n=0}^{\infty} \frac{a^n}{n!} x^n$$

is $-\infty < x < \infty$.

[5] Find the Maclaurin series expansion of the following functions using the Maclaurin series expansion in Problem [1].

(a) $f(x) = 1/(1+x)$

(b) $f(x) = 1/(1-x^2)$