

Fundamental Mathematics for Robotics

Handout #1 Trigonometric Identities

1. Sum of Angles (Addition Law)

- a. $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
- b. $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
- c. $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- d. $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

2. Double Angle

- a. $\sin(2x) = 2\sin(x)\cos(x)$
- b. $\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

3. Half Angle

- a. $\sin^2(x/2) = \{1 - \cos(x)\}/2$
- b. $\cos^2(x/2) = \{1 + \cos(x)\}/2$

4. Product to Addition

- a. $2\sin(x)\cos(y) = \sin(x + y) + \sin(x - y)$
- b. $2\cos(x)\sin(y) = \sin(x + y) - \sin(x - y)$
- c. $2\cos(x)\cos(y) = \cos(x + y) + \cos(x - y)$
- d. $2\sin(x)\sin(y) = -\cos(x + y) + \cos(x - y)$

5. Addition to Product

- a. $\sin(x) + \sin(y) = 2\sin \{(x + y)/2\} \cdot \cos \{(x - y)/2\}$
- b. $\sin(x) - \sin(y) = 2\cos \{(x + y)/2\} \cdot \sin \{(x - y)/2\}$
- c. $\cos(x) + \cos(y) = 2\cos \{(x + y)/2\} \cdot \cos \{(x - y)/2\}$
- d. $\cos(x) - \cos(y) = -2\sin \{(x + y)/2\} \cdot \sin \{(x - y)/2\}$

6. Other Identities

- a. $\sin(-x) = -\sin(x)$
- b. $\cos(-x) = \cos(x)$
- c. $\sin(x + \pi/2) = \cos(x)$
- d. $\cos(x + \pi/2) = -\sin(x)$
- e. $\sin(x + \pi) = -\sin(x)$
- f. $\cos(x + \pi) = -\cos(x)$
- g. $\sin^2(x) + \cos^2(x) = 1$
- h. $e^{jx} = \cos(x) + j \cdot \sin(x)$, where $j = \sqrt{-1}$