May 15, 2021, Dr.T

Fundamental Mathematics for Robotics

Handout #2 Rules of Differentiation and Integration

1. Differentiation Rules:
   (a) Linearity: $\frac{d}{dx}\{\alpha f(x) + \beta g(x)\} = \alpha \frac{d}{dx} f(x) + \beta \frac{d}{dx} g(x)$
   (b) Power: $\frac{d}{dx} x^n = nx^{n-1}$, $\frac{d}{dx} \ln|x| = \frac{1}{x}$
   (c) Chain Rule: $\frac{d}{dt} f(g(t)) = \frac{df(g)}{dg} \cdot \frac{dg(t)}{dt}$
   (d) Differentiation of product: $\frac{d}{dt}\{f(t)g(t)\} = \frac{df(t)}{dt} g(t) + f(t) \frac{dg(t)}{dt}$
   (e) Exponential: $\frac{d}{dt} e^{at} = \mathrm{a}e^{at}$
   (f) Sinusoidal function 1: $\frac{d}{dt} \sin \omega t = \omega \cos \omega t$
   (g) Sinusoidal function 2: $\frac{d}{dt} \cos \omega t = -\omega \sin \omega t$
   (h) $\frac{d}{dt} t^n e^{at} = nt^{n-1} e^{at} + at^n e^{at}$
   (i) $\frac{d}{dt} e^{\alpha t} \sin \omega t = \alpha e^{\alpha t} \sin \omega t + \omega e^{\alpha t} \cos \omega t$
   (j) $\frac{d}{dt} e^{\alpha t} \cos \omega t = \alpha e^{\alpha t} \cos \omega t - \omega e^{\alpha t} \sin \omega t$

2. Integration Rules:
   (a) Linearity: $\int\{\alpha f(x) + \beta g(x)\}dx = \alpha \int f(x)dx + \beta \int g(x)dx$
   (b) Power: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
   $n = -1$ -> Natural Logarithm: $\ln(x) = \int_1^x \frac{1}{u} du$, $x > 0$, $\int \frac{1}{x} dx = \ln(x) + C$
   (c) Chain Rule: $\int f(g(x)) \frac{dg(x)}{dx} dx = \int f(y)dy$ (substitution of y=g(x))
   (d) Integration by Parts: $\int f(x) \frac{dg(x)}{dx} dx = f(x)g(x) + C - \int \frac{df(x)}{dx} g(x)dx$
   (e) Exponential: $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$
   (f) Trigonometric functions: $\int \cos \omega x \, dx = \frac{1}{\omega} \sin \omega x + C$, $\int \sin \omega x \, dx = -\frac{1}{\omega} \cos \omega x + C$
   (g) $\int_a^b f(x)dx = -\int_b^a f(x)dx$
   (h) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$